CT vs DT signals

Tuesday, January 4, 2022 1:55 PM

Continous time: $x(t), y(t), z(t)$: $t \in \mathbb{R} \to \mathbb{C}$

Discrete time: $x[n], y[n], z[n]$: $n \in \mathbb{Z} \to \mathbb{C}$

Refresher: $c = a + bj = re^{j\theta}$

where $r = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

or $a = r\cos\theta$, $b = r\sin\theta$

Basic Operations on Signals

Wednesday, January 5, 2022 6:18 PM

Time Shift

$x(t) \to x(t - t_0) \quad t_0 \in \mathbb{R}$

$x[n] \to x(n - n_0) \quad n_0 \in \mathbb{Z}$

$t_0 / n_0 > 0$: delay

$t_0 / n_0 < 0$: advance

Time Reversal

$x(t) \to x(-t)$

$x[n] \to x[-n]$

Time Scale

$x(t) \to x(At)$

$x[n] \to x[An]$

$A > 1$: decimation

$A < 1$: expansion

↳ 0 when An is not an integer

Periodic, Energy Power, Even Odd Signals

Monday, January 10, 2022 7:53 PM

Periodic Signals: CT: $x(t)$ is periodic iff there exist a T such that $x(t+T) = x(t)$

DT: $x[n]$ is periodic iff there exists a N such that $x[n+N] = x[n]$

Fundamental Period: smallest T or N that is a period of a signal, T_0 or N_0

ω_0 (fundamental frequency) $= \frac{2\pi}{T_0}$ or $\frac{2\pi}{N_0}$

Energy / Power of a Signal:

$$E = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2$$

Even / Odd:

Even: $x(-t) = x(t)$ $\quad x[-n] = x[n]$

Odd: $x(-t) = -x(t)$ $\quad x[-n] = -x[n]$

Decomposition Theorem

Tuesday, January 11, 2022 2:11 PM

Every CT Signal $x(t)$ can be represented as:

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(t) = \dfrac{x(t) + x(-t)}{2}$, $x_o(t) = \dfrac{x(t) - x(-t)}{2}$

Every DT Signal $x[n]$ can be represented as:

$$x[n] = x_e[n] + x_o[n]$$

where $x_e[n] = \dfrac{x[n] + x[-n]}{2}$, $x_o[n] = \dfrac{x[n] - x[-n]}{2}$

Unit Impulse Signal, Complex Exponential Signals

Wednesday, January 12, 2022 5:46 PM

Unit impusle signal: $\delta(t)$, $\delta[n]$

Complex exponential signals: $ce^{\alpha t}$, ce^{an} where $c, a \in \mathbb{C}$

<u>Use</u> We can represent signals as linear combinations of unit impulse signals and complex exponential signals.

Therefore, superposition applies

<u>Def</u> $x[n] = e^{j\Omega n}$ is periodic iff Ω is a rational multiple of 2π

thus $x[n] = e^{j\frac{m2\pi}{N}n}$ where $N = \frac{2\pi m}{\Omega}$ is the fundamental period

DT Impulse, Step Signal

Wednesday, January 12, 2022 5:55 PM

Def DT Impulse and Unit step:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Def DT step signal:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Def We can define $u[n]$ in terms of $\delta[n]$:

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$, let $\ell = n-k$ then $u[n] = \sum_{\ell=-\infty}^{n} \delta[\ell]$

$$u[n] = \sum_{\ell=-\infty}^{n} \delta(\ell)$$ which is like a DT "integral" of $\delta[\ell]$ from $\ell = -\infty \to n$

$$u[n] = \sum_{k=-\infty}^{\infty} u[k]\,\delta[n-k]$$ thus for any $x[n]$: $\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}$

Def We can define $\delta[n]$ from $u[n]$: $\delta[n] = u[n] - u[n-1]$

CT Impulse, Step Signal

Monday, January 17, 2022 2:36 PM

Def CT unit step function: $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$

Def CT impulse function: $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{else} \end{cases}$ such that $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

Thus: $\int_{-\infty}^{t} \delta(T)\,dT = u(t)$

Properties of Delta Function

Wednesday, January 12, 2022 6:11 PM

Properties of $\delta[n]$:

1) Sampling Property: $x[n] \cdot \delta[n] = x[0] \cdot \delta[n]$

$$x[n] \cdot \delta[n-n_0] = x[n_0] \cdot \delta[n-n_0]$$

2) Sifting Property: $\sum_{n=-\infty}^{\infty} x[n] \cdot \delta[n] = x[0]$

$$\sum_{n=-\infty}^{\infty} x[n] \cdot \delta[n-n_0] = x[n_0]$$

3) Representation property ~~of~~ $\delta[n]$: for any DT signal $x[n]$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

Properties of $\delta(t)$:

1) Sampling Property: $x(t) \cdot \delta(t-T) = x(T) \cdot \delta(t-T)$

2) Shifting Property: $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-T)\, dt = x(T)$

3) Representation Property: $x(t) = \int_{-\infty}^{\infty} x(T)\,\delta(t-T)\, dT$

Systems, Properties of Systems

Wednesday, January 19, 2022 5:25 PM

Def Systems are a mapping from input signals to output signals

Can be CT → CT or DT → DT

Can be actual system or mathematical system

Properties:

1) Memoryless: output at time t/n only depends on input at t/n
2) BIBO Stability: output is bounded given a bounded input
3) Causal: output at time t/n only depends on input at $s \leq t/n$
4) Invertible: distinct inputs create distinct outputs, mapping is one-to-one
5) Time Invariant: if $x_1(t) \to y_1(t)$ then $x_1(t - t_0) \to y_1(t - t_0)$ iff system is Time Invariant
6) Linear: if $x_1 \to y_1$ $x_2 \to y_2$ then $ax_1 + bx_2 \to ay_1 + by_2$ iff system is Linear

Causal LTI System

Monday, January 24, 2022 4:06 PM

Def: Causal LTI system defined by difference or differential equations

In DT: $$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k]$$

In CT: $$\sum_{k=0}^{N} a_k \frac{\partial^k y(t)}{\partial t^k} = \sum_{k=0}^{M} b_k \frac{\partial^k x(t)}{\partial t^k}$$

when $N \geq 1$, the equation is called recursive.

when $N = 0$, the equation is called non-recursive.

Solution: $y[n] = y_p[n] + y_h[n]$

$y(t) = y_p(t) + y_h(t)$

"particular" + "homogenous"

$y_h[n]$ is the solution to $\sum_{k=0}^{N} a_k\, y[n-k] = 0$

$y_h(t)$ is the solution to $\sum_{k=0}^{N} a_k \frac{\partial^k y(t)}{\partial t^k} = 0$

for exact solution, we need starting conditions to solve for coefficients.
we specify that the system is causal and LTI

auxiliary condition is condition of initial rest:

- If $x[n] = 0$ for $n < n_0$, then $y[n] = 0$ for $n < n_0$
- If $x(t) = 0$ for $n < n_0$, then $y[n] = 0$ for $n < n_0$

Impulse Response: when $x[n] = \delta[n]$ or $x(t) = \delta(t)$, the output:

$y[n]$ or $y(t)$, we call this the impulse response, $h[n]$ or $h(t)$

DT Convolution Sum

Monday, January 24, 2022 5:59 PM

Def Given any input $x[n]$:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \quad \text{by representation property}$$

Given the impulse response $h[n]$ ~~of~~ an LTI system:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \quad \text{which we write as} \quad x[n] * h[n]$$

Def Computing $x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

1) set k as dependent variable, fix time n as constant
2) $h[n-k]$ as function of k is flipped and shifted right
3) multiply $x[k]$ to $h[n-k]$ and perform summation.

Properties:

1) $x[n] * \delta[n-n_0] = x[n-n_0]$
2) Distributivity: $(a+b)*(c+d) = a*c + a*d + b*c + b*d$ etc.
3) Associativity: $a*b*c = a*(b*c) = (a*b)*c$

CT Convolution Integral

Wednesday, January 26, 2022 4:50 PM

__Def__ Given some $x(t)$:

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\Delta\delta_\Delta(t-k\Delta) = \int_{-\infty}^{\infty} x(t)\delta(t)\,dt$$

Given the impulse response of an __LTI system__ $h(t)$:

$$y(t) = \int_{-\infty}^{\infty} x(T)h(t-T)\,dT = x(t) * h(t)$$

__Def__ Computing $x(t) * h(t) = \int_{-\infty}^{\infty} x(T)h(t-T)\,dT$

1) set k as dependent variable, fix time n as constant
2) $h[n-k]$ as function of k is flipped and shifted right
3) multiply $x[k]$ to $h[n-k]$ and perform summation.

__Properties__:

1) $x(t) * \delta(t-t_0) = x(t-t_0)$
2) Distributivity: $(a+b)*(c+d) = a*c + a*d + b*c + b*d$ etc.
3) Associativity: $a*b*c = a*(b*c) = (a*b)*c$

Properties of Causal LTI Systems from Impulse Response

Thursday, January 27, 2022 2:00 PM

1) Memoryless: memoryless iff $h[n] = a\delta[n]$ for $a \in \mathbb{C}$

memoryless iff $h(t) = a\delta(t)$ for $a \in \mathbb{C}$

2) Causal: causal iff $h[n] = 0$ for $n < 0$

causal iff $h(t) = 0$ for $t < 0$

3) Invertible: invertible iff $g[n] * h[n] = \delta[n]$ for any $g[n]$

invertible iff $g(t) * h(t) = \delta(t)$ for any $g(t)$

4) Stable: BIBO stable iff $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

BIBO stable iff $\int_{-\infty}^{\infty} |h(t)| < \infty$

Eigenfunction, Transfer Function, Frequency Response,

Monday, January 31, 2022 5:39 PM

Def Given some system $h(t)$ and a family of inputs $x_k(t)$ such that:

$$x_k(t) \rightarrow h(t) \rightarrow \lambda_k x_k(t)$$

then $x_k(t)$ is an eigenfunction and λ_k is an eigenvalue of the system

Idea Given some arbitrary $x(t)$ such that:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k x_k(t)$$

then $$x(t) \rightarrow h(t) \rightarrow \sum_{k=-\infty}^{\infty} \lambda_k a_k x_k(t)$$

Def Given a system $h(t)$ and input $x(t) = e^{st}$ for $s \in \mathbb{C}$

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} \, d\tau \qquad \text{and} \qquad y(t) = H(s) x(t)$$

$H(s)$ is the transfer function of the system

when $s = j\omega$ for $\omega \in \mathbb{R}$ then $H(s)$ is the frequency response

Def Given a system $h[n]$ and input $x[n] = z^n$ for $z \in \mathbb{C}$

$$H(z) = \sum_{k=-\infty}^{\infty} h[k] z^{-k} \qquad \text{and} \qquad y[n] = H(z) x[n]$$

$H(z)$ is the transfer function of the system

when $z = e^{j\omega}$ then $H(s)$ is the frequency response

Continuous Time Fourier Series

Monday, January 31, 2022 6:03 PM

Def for any $x(t)$ with finite energy over one period:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$$

Note: $\omega_0 = \frac{2\pi}{T}$

Notation: $x(t) \xleftrightarrow{F.S.} a_k$

Properties: given $x(t) \longleftrightarrow x_k$ and $y(t) \longleftrightarrow y_k$

1) Linearity: $ax(t) + by(t) \longleftrightarrow a x_k + b y_k$

2) Time Shift: $x(t-t_0) \longleftrightarrow x_k e^{-jk\omega_0 t_0}$

3) Time Reversal: $x(-t) \longleftrightarrow x_{-k}$

4) Time Scale: $x(at) \longleftrightarrow x_k$ (does not change)

5) Multiplication: $x(t)\cdot y(t) \longleftrightarrow x_k * y_k$

6) Conjugation: $x^*(t) \longleftrightarrow a^*_{-k}$

7) Parseval: $\frac{1}{T}\int_T |x(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |a_k|^2$

Discrete Time Fourier Series

Wednesday, February 2, 2022 7:03 PM

Def for any $x[n]$ with finite energy over one period:

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} \quad \text{where} \quad a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n}$$

Note: $\omega_0 = \frac{2\pi}{N}$ and $N \geq 1$ and $a_{k+N} = a_k$

Notation: $x[n] \xleftrightarrow{F.S.} a_k$

Properties: given $x[n] \longleftrightarrow x_k$ and $y[n] \longleftrightarrow y_k$

1) Linearity: $a x[n] + b y[n] \longleftrightarrow a x_k + b y_k$

2) Time Shift: $x[n - n_0] \longleftrightarrow x_k e^{-jk\omega_0 n_0}$

3) Time Reversal: $x[-n] \longleftrightarrow x_{-k}$

4) Time Scale: $x[at] \longleftrightarrow x_k$ (does not change)

5) Multiplication: $x[n] \cdot y[n] \longleftrightarrow x_k * y_k$

6) Conjugation: $x^*[n] \longleftrightarrow a^*_{-k}$

7) Parseval: $\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$

Solving Output of LTI Given FS and H(s)

Tuesday, February 8, 2022 2:17 PM

Def Given some periodic input and the impulse response of LTI system, we can find the output. Assume fundamental frequcey is ω_0.

CT: If $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ and $H(jk\omega_0) = \int_{-\infty}^{\infty} h(T) e^{-jk\omega_0 T} \, dT$

then $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$ where $b_k = a_k \cdot H(jk\omega_0)$

DT: If $x[n] = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 n}$ and $H(e^{jk\omega_0}) = \sum_{N=-\infty}^{\infty} h[N] e^{-jk\omega_0 N}$

then $y[n] = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 n}$ where $b_k = a_k \cdot H(e^{jk\omega_0})$

Steps:
1) find input as a FS $\rightarrow a_k$
2) find impulse response as a function of $k\omega_0$
3) multiply each a_k by impulse response $\rightarrow b_k$
4) build FS representation of output using b_k

Continuous Time Fourier Transform

Wednesday, February 9, 2022 5:15 PM

__Def__ Given some non-periodic signal $x(t)$:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \partial\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \partial t$$

__Def__ Given some periodic signal $x(t)$ with fundamental frequency ω_0:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where a_k are the fourier series coefficients

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

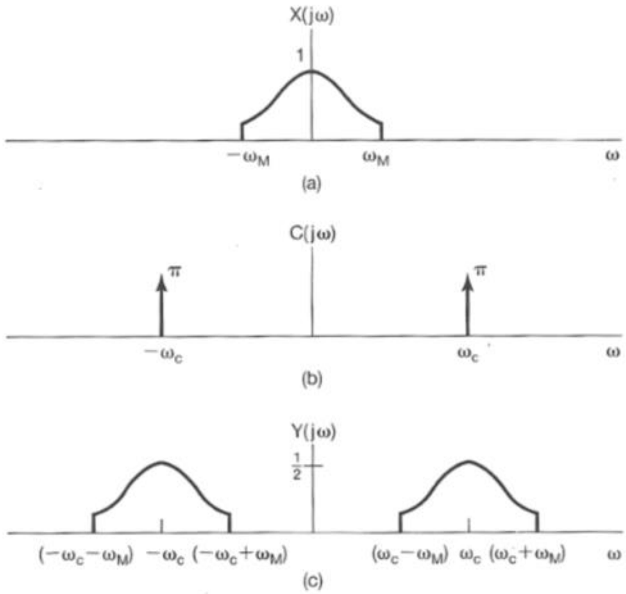
__Notation__: $x(t) \xleftrightarrow{F.T.} X(j\omega)$

Amplitude Modulation

Thursday, February 17, 2022 2:01 PM

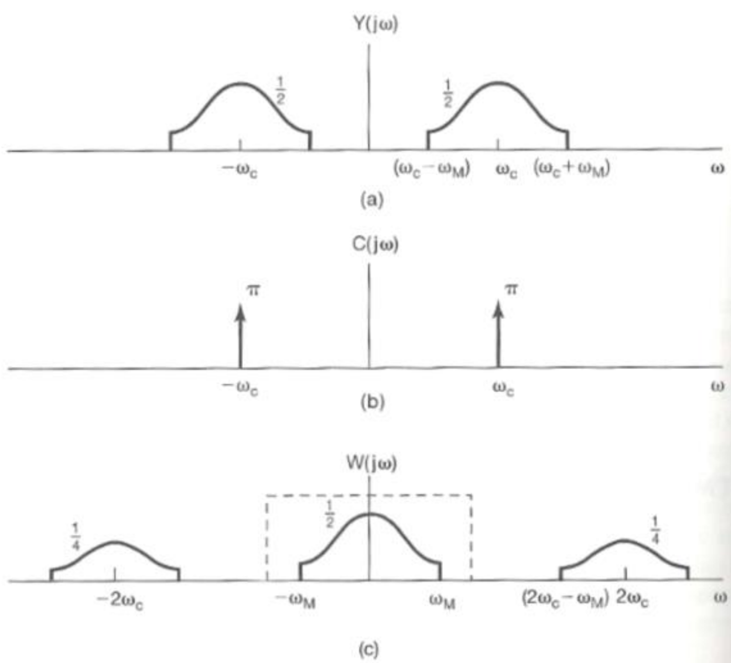
<u>Def</u> If we have some signal $x(t)$ and a carrier signal $c(t) = \cos(\omega_0 t)$

if $x(t) \longleftrightarrow X(j\omega)$ then $x(t) * c(t) \longleftrightarrow \frac{1}{2}\left[X(j(\omega-\omega_0)) + X(j(\omega+\omega_0))\right]$



where $y(t) = x(t) * c(t)$ is the modulated signal

<u>Def</u> To demodulate the signal, apply $y(t) * c(t)$ and use a low pass filter. Then multiply by 2:

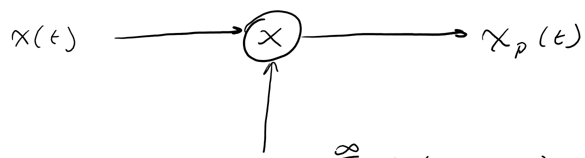


where $W(j\omega)$ is now $\frac{1}{2}$ the spectrum of the original signal $X(j\omega)$.

Sampling Theorem

Monday, February 21, 2022 4:27 PM

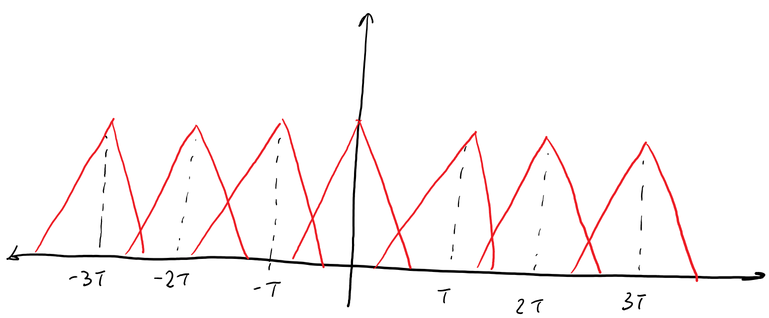
<u>Def</u> Given a CT signal, if we sample the signal with a train of impulse signals:



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

we can reconstruct the original signal iff $x(t)$ has limited bandwidth of size $\pm W$ and $T \geq 2W$

<u>Def</u> The spectrum of a signal sampled with period T:



Any overlap is called <u>Aliasing</u>

Discrete Time Fourier Transform

Wednesday, February 23, 2022 5:08 PM

Def Given some aperiodic $x[n]$:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Def Given some periodic signal $x(t)$ with fundamental frequency ω_0:

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n}$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

where a_k are the fourier series coefficients and N is the fundamental period

Notation: $x[n] \xleftrightarrow{F.T.} X(e^{j\omega})$

Processing CT in Discrete Time

Monday, February 28, 2022 5:03 PM

<u>Def</u> Given some input $x(t)$ we can process it by:

1) Converting $x(t) \rightarrow x[n]$ by sampling with period T
2) Process $x[n]$ using some system $h[n]$ or $H(e^{j\omega})$ to get $y[n]$
3) convert $y[n] \rightarrow y(t)$ by applying a low pass filter to $Y(e^{j\omega})$

<u>Procedure</u>: given $x(t)$ the input and $p(t)$ an impulse train with period T:

1) $x_p(t) = x(t)p(t) \longleftrightarrow X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\tfrac{2\pi}{T}))$$

2) $x_d[n] = x(nT) \longleftrightarrow X_d(e^{j\Omega}) = X_p(j\Omega/T)$ where $\Omega = \omega T$

3) apply filters: $y_d[n] = x_d[n] * h[n] \longleftrightarrow Y_d(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$

4) $y_p(t) = y(nT) \longleftrightarrow Y_p(j\omega) = Y_d(e^{j\omega})$ where $\omega = \frac{\Omega}{T}$

5) $y_c(t) = y_p(t) * h_{LP}[n] \longleftrightarrow Y_c(j\omega) = Y_p(j\omega) * H_{LP}(j\omega)$

Laplace Transform

Wednesday, March 2, 2022 4:23 PM

<u>Idea</u> CTFT and DTFT only apply to finite energy signals and stable LTI systems

We can use Laplace Transforms to analyze non finite energy signals

<u>Def</u> For a continous time signal $x(t)$:

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, \partial t \qquad \text{for } s = \sigma + j\omega$$

The values of s for which $\mathcal{L}\{x(t)\}$ exists is the ROC

<u>Existence</u>: $LT\{x(t)\}$ exists iff $CTFT\{x(t)e^{-\sigma t}\}$ exists

if $s = j\omega$ then $LT\{x(t)\}$ exists iff $CTFT\{x(t)\}$ exists

if $x(t)$ is absolutely integrable, then ROC contains $j\omega$ axis

<u>Def</u> Given $\mathcal{L}\{x(t)\} = X(s)$ then:

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} X(s)\, e^{st}\, \partial s$$

Common Laplace Transform Pairs

Thursday, March 3, 2022 1:54 PM

<u>Common $\mathcal{L}$ pairs</u>:

$$e^{-at}\,u(t) \longleftrightarrow \frac{1}{s+a} \quad \text{for } Re\{s\} > -Re\{a\}$$

$$-e^{-at}\,u(-t) \longleftrightarrow \frac{1}{s+a} \quad \text{for } Re\{s\} < -Re\{a\}$$

Properties of Laplace Transform

Thursday, March 3, 2022 1:58 PM

Table 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Property	Signal	Laplace Transform	ROC
	$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
Linearity	$a\,x_1(t) + b\,x_2(t)$	$a\,X_1(s) + b\,X_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-s t_0} X(s)$	R
Shifting in s	$e^{s_0 t}\,x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(a\,t)$	$\frac{1}{\|a\|} X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)\,X_2(s)$	At least $R_1 \cap R_2$
Differentiation	$\frac{d}{dt}x(t)$	$s\,X(s)$	At least R
Differentiation in s	$-t\,x(t)$	$\frac{d}{ds}X(s)$	R
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathfrak{Re}\{s\} > 0\}$

Initial Value Theorem:

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then $x(0^+) = \lim_{s\to\infty} s\,X(s)$.

Final Value Theorem:

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \to \infty$, then $\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s)$.

Zeros and Poles. ROC Constraints

Thursday, March 3, 2022 2:05 PM

Def Given $X(s)$ which is rational:

$$X(s) = \frac{N(s)}{D(s)}$$ where $N(s)$ is the zeros of $X(s)$
$D(s)$ is the poles of $X(s)$

where $N(s) = \prod (s - n_i)$ and $D(s) = \prod (s - d_i)$

Def The ROC cannot contain any poles
If $x(t)$ is absolutely integrable, ROC includes $j\omega$ axis.
If $x(t)$ is a finite signal, ROC is the entire s plane

Def A signal is right sided iff ROC $> s = k$ for some $k \in \mathbb{R}$
A signal is left sided iff ROC $< s = k$ for some $k \in \mathbb{R}$